Renormalization-group analysis of the smectic-A_1–smectic-A_d phase transition in liquid crystals

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The present paper develops the renormalization-group techniques to analyze the phase transition between the smectic-A_1 and smectic-A_d phases of liquid crystals. This transition exhibits a bicritical or a tetracritical point, depending upon the number of components (n) of the order parameter. For $n \leq 3$ an isotropic or Heisenberg fixed point dominates and gives bicritical behavior. However, for $n \geq 4$ a new fixed point, with irrational ε expansion coefficients, becomes stable and describes tetracritical behavior. The critical exponents are calculated to first order in ε.

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INTRODUCTION

In the smectic-A (Sm-A) phase of liquid crystals [1] oriented barlike molecules of length L segregate into stacks of structureless two-dimensional planes. It is now clear that there are several types of Sm-A phases [2,3] characterized by the ratio of the interplanar spacing d to L. The conventional Sm-A phases have been classified according to the periodicity. When it corresponds to the molecular length L, the phase is called a monolayer or Sm-A_1 (A_1) phase, and when the periodicity corresponds to twice the molecular length, $2L$, the phase is called a bilayer or Sm-A_2 (A_2) phase. There are numerous "conventional" examples of both monolayer and bilayer smectic systems that undergo continuous, or second-order, transitions between the nematic (N) and Sm-A phases for which the critical density fluctuations exhibit only a single wave vector corresponding to either the monolayer or bilayer periodicity. Again a new class of polar liquid crystal molecules has been discovered, which exhibit a variety of new smectic phases including polar monolayer and bilayer phases and a Sm-A_d (A_d) phase with a periodicity d intermediate between the monolayer and bilayer periodicity; $L<d<2L$. These remarkable polar Sm-A phases melt into polar nematic phases which exhibit two simultaneous fluctuations with commensurate $d_1=2d_2$. In this paper I shall be concerned with the A_d-A_1 transition.

Sm-A polymorphism has been successfully explained in terms of a phenomenological model [4–7]. In the paper of Barois, Prost, and Lubensky [8] this model was used in the framework of mean-field theory to evaluate different types of phase diagrams. They predicted that a bicritical point exists in the phase diagram, where the second-order (N_{re}-A_1) and (N_{re}-A_d) phase boundaries meet a first-order A_d-A_1 line. Here N_{re} represents the reentrant nematic phase. This is directly analogous to the magnetic bicritical point. However, when the effect of fluctuations is considered in the theory, the existence of such a bicritical point becomes questionable. It has been argued [8] that since both A_d and A_1 phases have the same symmetry the N_{re}-A_d and N_{re}-A_1 transitions should both belong to the same universality class [9]. Further renormalization-group calculations show [10] that under such circumstances the resulting multicritical point should be a tetracritical point and not a bicritical point.

Furthermore, experimentally the A_d-N_{re}-A_1 point has been observed in the temperature-concentration diagram of binary liquid crystal systems [11,2]. Again, a high resolution phase diagram for $DB_8ONO_2+DB_{10}ONO_2$ mixtures [12] [DB_8ONO_2 is 4-n-octyloxyphenyl-4′-(4″-nitrobenzolyloxy) benzoate and $DB_{10}ONO_2$ is 4-n-decyloxyphenyl-4′-(4″-nitrobenzoyloxy) benzoate] shows that the topology near the A_d-N_{re}-A_1 point does not conform to the expected bicritical or tetracritical point.

Indeed, it seems likely that the full classification of multicritical points will, like the classification of knots, remain an esoteric and largely unsolved problem for some time. For the present it thus seems reasonable to proceed in a more frankly *ad hoc* fashion and investigate various multicritical points as they come to hand in significant contexts. Multicritical points can be defined phenomenologically as points of sudden change of behavior on a line of critical points (i.e., a second-order transition line). A step in this direction was taken by Liu and Fisher [13] who presented a phenomenological analysis of the multicritical points resulting from the competition between two distinct types of ordering.

When the interaction is isotropic, the Hamiltonian of ferromagnets may be described by the Heisenberg model

$$H=-J\sum_{\langle ij\rangle}(\mathbf{S}_i\cdot\mathbf{S}_j) . \quad (1)$$

Real crystals never obey the idealized Hamiltonian (1). The coupling to the lattice degrees of freedom usually breaks the rotational symmetry, and generates easy axes along which the spins align. For example, a spatial uniaxial anisotropy of the lattice may generate, via the spin-orbit coupling, a uniaxial anisotropy in the spins, via single ion terms like

$$\beta H_\alpha=\tfrac{1}{2}g\sum_i\{(S_i^z)^2-\tfrac{1}{2}[(S_i^x)^2+(S_i^y)^2]\} . \quad (2)$$

Similar terms may be generated experimentally by the application of uniaxial stress (proportional to g). For $g<0$, the Hamiltonian (2) has preferential ordering of the spins along the z axis. For $g>0$, the preferential ordering is in the XY planes. At low temperatures, this competition yields a first-order "spin flop" transition at $g=0$, at which the magnetization rotates discontinuously by 90°. The complete T-g phase diagram is thus expected to have the qualitative shape shown in Fig. 1; the ordering is along the z axis for $g<0$ and in the XY plane for $g>0$. For large $|g|$, the fluctuations in the transverse XY plane (when $g<0$) are expected to be negligible. The transition for $g<0$ is thus expected to exhibit critical properties characteristic for the Ising ($n=1$) model. Similarly, the transition for $g>0$ is expected to exhibit XY ($n=2$) critical behavior. The point at which these two lines meet, at $g=0$, is called a bicritical point [14,15]. So a bicritical point may be characterized as the meeting of two separate critical lines corresponding to two distinct order parameters. It is only at this point that one expects to observe the critical behavior of the "true" Heisenberg ($n=3$) model Eq. (1).

In addition to the (quadratic) uniaxial anisotropy (2), real systems usually also have higher order symmetry breaking interactions. In cubic systems, one expects the single ion cubic Hamiltonian

$$\beta H_c = v \sum_i \sum_{\alpha=1}^{n} (S_i^\alpha)^4 \ . \tag{3}$$

Such a term has preferential ordering of the spins along cubic axes (e.g., [100]) if $v<0$, and along cubic diagonals (e.g., [111]) if $v>0$.

When both the uniaxial anisotropy, Eq. (2), and the cubic one, Eq. (3), arise simultaneously, competition between them may occur. Indeed, when $v>0$ then Eq. (3) prefers ordering along diagonals while Eq. (2) prefers ordering along axes. The resulting phase diagram is shown in Fig. 2: the flop line is now replaced by two second-order lines, and the multicritical point is now called a "tetracritical point" [16]. In particular, at tetracritical points four critical lines meet. In fact, one may show that the tetracritical and the bicritical point are the same point, when viewed in a larger parameter space [17].

In the present paper I have used the renormalization-group technique to investigate these multicritical points of the A_1-A_d phase transition. I find that bicriticality is attained only at an isolated fixed point. All the exponents associated with this fixed on it are just those of the usual n-isotropic, Heisenberg model. I find that within the

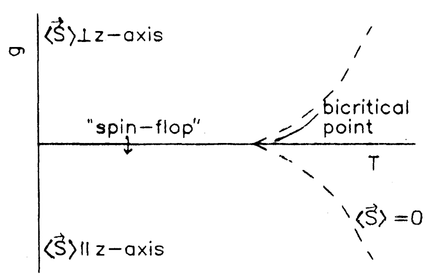

FIG. 1. Bicritical point. The solid line is the first-order transition line and the dashed lines are second-order transition lines.

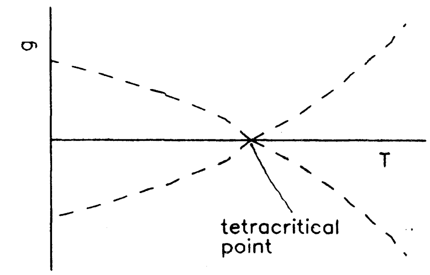

FIG. 2. Tetracritical point. The dashed lines are second-order transition lines.

scaling regime tetracritical behavior should not be realizable for $n<n^*(d)$.

THEORY

In the smectic-A phase, the center-of-mass density ρ can be expanded in a Fourier series of period d, the interplanar spacing,

$$\rho=\rho_0+\sum_{n>0}\rho_n\cos[n(\mathbf{q}_0\cdot\mathbf{r}+\psi)]$$

$$=\rho_0+\tfrac{1}{2}\sum_{n>0}(\rho_n e^{in\psi}e^{in\mathbf{q}_0\cdot\mathbf{r}}+\text{c.c.}) \ . \tag{4}$$

$\mathbf{q}_0$ is a vector of magnitude $q_0=2\pi/d$ normal to the smectic phase and ψ is a phase shift which specifies the coordinate-system ordering. One parameter representing unambiguously the vectorial symmetry of molecules is, for example,

$$\mathbf{P}(\mathbf{r})=\frac{1}{v}\sum_i \mathbf{P}_i\delta(\mathbf{r}-\mathbf{r}_i) \ , \tag{5}$$

where the summation is performed inside the volume v and $\mathbf{P}_i$ is the dipole of molecule i. This parameter describes the antiferroelectric order which tends to condense at the wavelength of molecular pairs [4]. In most of the following, one can make the simplifying choice of potential where, in the absence of charges, $\mathbf{P}$ is derived as order parameter:

$$\mathbf{P}(\mathbf{r})=\frac{1}{4\pi}\nabla\phi \ . \tag{6}$$

The mass density, of course, is a relevant order parameter, assuming that it tends to condense at a wavelength corresponding to the molecular length. Hence a good choice of order parameters would be

$$\phi=(\rho_A-\rho_B)/2 \ , \quad \rho=(\rho_A+\rho_B)/2 \ , \tag{7}$$

in which one can distinguish part A from part B in the molecule (say, head and tail) and ρ_A and ρ_B, respectively, are the densities of parts A and B. Again, ρ can be assumed to condense at the molecular length; ϕ describes the segregation between heads and tails and thus can be assumed to condense at the pair length. ϕ will be called the antiferroelectric order parameter in what follows.

The simplest model free energy F capable of describing the nematic (N), Sm-A_1, and Sm-A_d phases is a functional field of the fields ρ and ϕ only, where ρ and ϕ are the order parameters of the Sm-A_1 and Sm-A_d phases. In general, of course, all ρ_n and ϕ_n are needed, but they can be expressed as functions of ρ and ϕ. The free energy must be invariant under uniform translations of the system. Mean-field diagrams involving A_d-$N_{\rm re}$-A_1 points do not exhibit reentrant behavior but they do show that reentrant terms of the free energy near that point are typical of two order parameters coupled only at fourth order [7],

$$F=\int d^d\times[\tfrac{1}{2}A_\rho\rho^2+\tfrac{1}{2}(\nabla\rho)^2+\tfrac{1}{2}A_\phi\phi^2+\tfrac{1}{2}(\nabla\phi)^2+\tfrac{1}{4}B_\rho\rho^4+\tfrac{1}{4}B_\phi\phi^4+\tfrac{1}{2}B_{\rho\phi}\rho^2\phi^2]\ . \tag{8}$$

Such a free energy has been extensively studied in d dimensions, ρ and ϕ being considered as vectorial order parameters with n components. In the smectic case, ρ and ϕ are two-component order parameters

$$\rho=\begin{bmatrix}\rho_0\cos\psi_\rho\\ \rho_0\sin\psi_\rho\end{bmatrix}\ ,\quad \phi=\begin{bmatrix}\phi_0\cos\psi_\phi\\ \phi_0\sin\psi_\phi\end{bmatrix}\ , \tag{9}$$

where, ψ_ρ and ψ_ϕ are the phases, ρ and ϕ are the monolayer and partial bilayer order parameters, and

$$A_\rho=a_\rho(T-T_0)\ , \tag{10}$$

$$A_\phi=a_\phi(T-T_0)\ . \tag{11}$$

The quartic coefficients B_ρ, B_ϕ, and $B_{\rho\phi}$ are positive, varying slowly with temperature, and a_ρ and a_ϕ are positive constants. T is the temperature and T_0 is the mean-field N-A_1 and N-A_d transition temperature. The equations neglect the coupling with the director fluctuation. The free energy of Eq. (8) has been used to study the $N_{\rm re}$-A_1, $N_{\rm re}$-A_d, and A_d-A_1 transitions theoretically [4–7]. A renormalization-group analysis can be applied directly to this model to study the A_1-A_d transition.

Hand-waving arguments [5] suggest that reentrance may occur when $A_\rho\simeq A_\phi<0$ but $A_\rho+B_{\rho\phi}\langle\phi^2\rangle$ and $A_\phi+B_{\rho\phi}\langle\rho^2\rangle$ are greater than zero. In other words, reentrance is induced by the fluctuations of the coupling order parameters. Solutions for quasi-one-dimensional Ising system involving the transfer integral technique [3] and the harmonic approximation for the coupled XY parameters [6], together with an exact solution of the $n=\infty$ case do reveal a reentrant behavior connected to the coexistence of a multicritical point (bicritical or tetracritical). The general argument can be cast in a simple form for any n and $2<d<4$. The experimental values of n in the case of the A_1-A_d transition is $n=2$.

Assuming finite anisotropy, a different analysis is needed in the bicritical region where $A_\rho\simeq A_\phi$ and also assuming A_ρ, A_ϕ, B_ρ, B_ϕ, and $B_{\rho\phi}$ are all of order $\varepsilon=4-d$. In this paper I present the results of an analysis of this model (8) based on Wilson's renormalization-group approach [18,19] for $d=4-\varepsilon$ and arbitrary n. We start from Wilson's recursion formula [20]

$$Q_{K+1}(Z)=-b^d\ln[I_K(b^{1-d/2}Z)/I_K(0)]\ , \tag{12}$$

$$I_K(Z)=\int_{-\infty}^{+\infty}dy_1\cdots\int_{-\infty}^{+\infty}dy_n\exp[-y^2-\tfrac{1}{2}Q_K(Y+Z)-\tfrac{1}{2}Q_K(-Y+Z)]\ . \tag{13}$$

The recursion relations are readily constructed to the leading order [18,21] and found to be

$$A'_\rho=b^2[A_\rho+3fB_\rho+f(n-1)B_{\rho\phi}-3gB_\rho A_\rho-g(n-1)B_{\rho\phi}A_\phi]\ , \tag{14}$$

$$A'_\phi=b^2[A_\phi+f(n+1)B_\phi+fB_{\rho\phi}-g(n+1)B_\phi A_\phi-gB_{\rho\phi}A_\phi]\ , \tag{15}$$

$$B'_\rho=b^\varepsilon[B_\rho-9gB_\rho^2-g(n-1)B_{\rho\phi}^2]\ , \tag{16}$$

$$B'_\phi=b^\varepsilon[B_\phi-g(n+7)B_\phi^2-gB_{\rho\phi}^2]\ , \tag{17}$$

$$B'_{\rho\phi}=b^\varepsilon B_{\rho\phi}[1-3gB_\rho-g(n+1)B_\phi-4gB_{\rho\phi}]\ , \tag{18}$$

where

$$f(b)=\Lambda^2(b^{-2}-1)/8\pi^2\ , \tag{19}$$

$$g(b)=\ln b/8\pi^2\Lambda^\varepsilon \tag{20}$$

arise from the usual Feynman type integrals over the outer momentum shell with cutoff Λ evaluated as $d\to4$. Here the prime denotes the superscript $(k+1)$, while on the right-hand side the superscript (k) has been dropped. To investigate the stability of the solutions obtained it is necessary to find the eigenvalues of the determinant of the linearized system of Eqs. (14)–(18). Here b is the momentum cutoff reduction factor $(b>1)$.

It is evident from the above recursion relations that for any value of $n(>0)$ the last three of these equations determine six fixed points. They are

$$B^*_{\rho\phi}=0\ ,\quad B^*_\phi=0\ ,\quad B^*_\rho=0\ , \tag{21a}$$

$$B^*_{\rho\phi}=0\ ,\quad B^*_\phi=0\ ,\quad B^*_\rho=16\pi^2\Lambda^\varepsilon\varepsilon/9\ , \tag{21b}$$

$$B^*_{\rho\phi}=0\ ,\quad B^*_\phi=16\pi^2\Lambda^\varepsilon\varepsilon/(n+7)\ ,\quad B^*_\rho=0\ , \tag{21c}$$

$$B^*_{\rho\phi}=0\ ,\quad B^*_\phi=16\pi^2\Lambda^\varepsilon\varepsilon/(n+7)\ ,\quad B^*_\rho=16\pi^2\Lambda^\varepsilon\varepsilon/9\ , \tag{21d}$$

$$B^*_\rho=B^*_\phi=B^*_{\rho\phi}=8\pi^2\Lambda^\varepsilon\varepsilon/(n+8)\ ,$$

with

$$A^*_\rho=A^*_\phi=-4\pi^2\Lambda^\varepsilon\varepsilon(n+2)/(n+8)\ , \tag{21e}$$

and

$$\begin{aligned}
B^*_\rho&=\{1+[1-9(n-1)x^2]^{1/2}\}4\pi^2\Lambda^\varepsilon\varepsilon/9\ ,\\
B^*_\phi&=\{1+[1-(n+7)x^2]^{1/2}\}4\pi^2\Lambda^\varepsilon\varepsilon/(n+7)\ ,\\
B^*_{\rho\phi}&=4\pi^2\Lambda^\varepsilon\varepsilon x\ ,\\
A^*_\rho&=[3fB^*_\rho+(n-1)fB^*_{\rho\phi}]/(b^{-2}-1)\ ,\\
A^*_\phi&=[(n+1)fB^*_\phi+fB^*_{\rho\phi}]/(b^{-2}-1)\ ,
\end{aligned} \tag{21f}$$

where x is the real root of the cubic equation

$$px^3-qx^2+rx+s=0 \tag{22}$$

with

$$p=9(4n^2+29n+88)\ ,\quad q=6(2n^2+28n+179)\ ,$$
$$r=(n^2+5n+472)\ ,\quad \text{and } s=6(n-11)\ .$$

DISCUSSION WITH RESULTS

Now the fixed point (21a) is trivial, always unstable, and a Gaussian-Gaussian point. The fixed point (21b) is an Ising-Gaussian point. The fixed point (21c) is a Gaussian $(n-1)$ Heisenberg point and the fixed point (21d) is a decoupled Ising $(n-1)$ Heisenberg fixed point. For $n<11+O(\varepsilon)$ all the fixed points from Eqs. (21a)–(21d) are found to be unstable to the $B_{\rho\phi}$ perturbation. The various crossover exponents associated with these in-plane flows are all of order ε. Calculating the renormalization-group eigenvalues corresponding to perturbations which take the system out of $B^*_{\rho\phi}=0$ leads to the eigenvalues

$$\lambda_1=\varepsilon\ ,\quad \lambda_2=\frac{2\varepsilon}{3}\ ,\quad \lambda_3=\frac{6\varepsilon}{n+7}\ ,\quad \lambda_4=\frac{(11-n)\varepsilon}{3(n+7)}\ , \tag{23}$$

where we have written the b-dependent renormalization eigenvalues $\Lambda_{<\infty}(b)$ as $\Lambda_{<\infty}=b^{\lambda(\alpha)}$. The first three fixed points are evidently unstable to $B_{\rho\phi}$-type perturbations for all $n>-8$. The fixed point (21d), however, is only unstable when

$$n<11+O(\varepsilon)\ . \tag{24}$$

If this inequality is reversed, the fixed point becomes completely stable and terminates the critical surface. Since the system will then spontaneously break into n Heisenberg essentially independent systems, a single scaling function cannot properly describe the asymptotic free energy when the values of n of Sm-A_d and Sm-A_1 transitions are different.

The fixed point (21e) is an isotropic n Heisenberg point. As the interaction parameters B_ρ, B_ϕ, and $B_{\rho\phi}$ at this fixed point satisfy the mean-field theory criterion for bicritical behavior [13], $(B^*_{\rho\phi})^2\geq B^*_\rho B^*_\phi$, we conclude that this fixed point describes a bicritical point. Linearizing about this fixed point in $(B_\rho,B_\phi,B_{\rho\phi})$ space, we find the three eigenvalues

$$\lambda'_1=-\varepsilon\ ,\quad \lambda'_2=-\frac{8\varepsilon}{n+8}\ ,\quad \lambda'_3=\frac{(n-4)\varepsilon}{(n+8)} \tag{25}$$

correct to order ε [21,22]. This fixed point is fully stable and hence determines the critical behavior for

$$n\leq 4+O(\varepsilon)\ . \tag{26}$$

Now using the eigenvalues λ'_3 obtained to $O(\varepsilon^3)$ by Ketley and Wallace [23], one can find that this fixed point remains stable in the full $(B_\rho,B_\phi,B_{\rho\phi})$ subspace for

$$n<n^*(d)=4-2\varepsilon+c^*\varepsilon^2+O(\varepsilon^3)\ ,$$

where $c^*=\frac{5}{12}[6\zeta(3)-1]$ takes the same form as in [13]. Now for $d=3$ this yields $n^*(3)\simeq 3.128$. Thus in three dimensions one still expects the Heisenberg fixed point to dominate for $n\leq 3$.

What transpires from the above analysis that both A_ρ and A_ϕ are specified at the fixed point indicates that bicriticality is attained only at isolated points. All the exponents associated with this fixed point are just those of the usual n-isotropic, Heisenberg model.

The value of the critical exponent γ of the susceptibility may be obtained in the usual way as

$$\gamma=1+\varepsilon\frac{(n+2)}{2(n+8)}\ . \tag{27}$$

The crossover index is given by [24]

$$\Phi=1+\varepsilon\frac{n}{2(n+8)}\ . \tag{28}$$

Now the fixed point of Eq. (21f) becomes stable and determines critical behavior for $n^*(d)<n<11+O(\varepsilon)$. In order to obtain the eigenvalues and also the critical exponents, the root of Eq. (22) is necessary. Although the appropriate root of this equation is rational at $n=11$ $(x=0)$, $n=4$ $(x=\frac{1}{6})$, $n=2$ $(x=\frac{1}{8})$, and at $n=1$ and -1 the root is an irrational function of n. For $n=5$, we have

$$x=\{82-[a+b(82)^{1/2}]^{1/3}-[a-b(82)^{1/2}]^{1/3}\}/333\ ,$$

where $a=18728$ and $b=1998$.

The renormalization-group eigenvalues, and hence the critical point exponents, can be calculated to order ε through (14)–(18) and again have irrational coefficients. To order ε this yields

$$\lambda''_1=2+\tfrac{1}{2}(-3B^*_\rho-(n+1)B^*_\phi+\{[3B^*_\rho-(n+1)B^*_\phi]^2+4(n-1)(B^*_{\rho\phi})^2\}^{1/2})\ ,$$
$$\lambda''_2=2+\tfrac{1}{2}(-3B^*_\rho-(n+1)B^*_\phi+\{[3B^*_\rho-(n+1)B^*_\phi]^2+4(n-1)(B^*_{\rho\phi})^2\}^{1/2})\ . \tag{29}$$

From these relations the thermodynamic exponents can be calculated using the standard expression [25–27]

$$2-\alpha=d\nu=d/\lambda''_1 \tag{30}$$

while the crossover exponent is given by [25–27]

$$\Phi=\lambda''_2/\lambda''_1\ . \tag{31}$$

The gap exponent Δ entering the free energy scaling relation [28] is related to η by

$$\Delta_\rho=\tfrac{1}{2}(d+2-\eta_\rho)\nu\ ,\quad \Delta_\phi=\tfrac{1}{2}(d+2-\eta_\phi)\nu\ , \tag{32}$$

and similarly the susceptibility exponents

$$\gamma_\rho=(2-\eta_\rho)\nu\ ,\quad \gamma_\phi=(2-\eta_\phi)\nu\ . \tag{33}$$

It is in fact possible to determine the tetracritical exponents η_ρ and η_ϕ to leading order by straightforward techniques [25–27]. The inequality of the fixed point values of A^*_ρ and A^*_ϕ as evidenced by (21f) leads to distinct exponents η_ρ and η_ϕ. To order ε^2 these are given by

$$\eta_\rho=8[3B^{*2}_\rho+(n-1)B^{*2}_{\rho\phi}]+O(\varepsilon^3)\ , \tag{34}$$

$$\eta_\phi=8[(n+1)B^{*2}_\phi+B^{*2}_{\rho\phi}]+O(\varepsilon^3)\ , \tag{35}$$

where only the fixed point values (29) of B_ρ^*, B_ϕ^*, and $B_{\rho\phi}^*$ to $O(\varepsilon)$ are needed.

In the region of stability, it can be easily shown that $0 \le B_{\rho\phi}^* < 32\pi^2\Lambda^\varepsilon\varepsilon/(n+8)$, while B_ρ^* and B_ϕ^* exceed $32\pi^2\Lambda^\varepsilon\varepsilon/(n+8)$. On further increase of n the solution (21f) also becomes unstable. Accordingly this fixed point then satisfies the condition $(B_{\rho\phi}^*)^2 < B_\rho^* B_\phi^*$ which represents the phenomenological criterion [13] for tetracriticality, i.e., a new intermediate phase exists with both Sm-A_d and Sm-A_1 order simultaneously present. The condition of bicriticality [13], namely, $(B_{\rho\phi}^*)^2 \ge B_\rho^* B_\phi^*$, is satisfied at the Heisenberg critical point so that, within the scaling regime, tetracritical behavior should not be realizable for $n < n^*(d)$.

When $n > 4$, the intersection point will be tetracritical, i.e., an intermediate phase arises. This intermediate phase is the Sm-A_d phase. However, I must distinguish two cases.

For $(n-8)(n+16) > n^2$ there is an uncoupled tetracritical fixed point. In this case the susceptibility and the crossover exponents are

$$\gamma_\rho = 1+\varepsilon/6 \ , \quad \gamma_\phi = 1+\frac{n+1}{2(n+7)}\varepsilon \ , \quad \text{and } \Phi = 1 \ . \qquad (36)$$

That is, the system at an "uncoupled" tetracritical point behaves like two noninteracting subsystems and the transition lines intersect at this point at an angle ($\rho = \phi = 1$).

When $4 < n < 8+(n-2)^2/(n+16)$, then $B_{\rho\phi}^* > 0$, $\rho > 1, \phi > 1$, and the transition lines are tangent to each other at the tetracritical point. The exponents also turn out to be different [29].

CONCLUSION

In conclusion, the present analysis clearly indicates that the critical exponents observed in these A_d-A_1 transitions provide theoretical support to the experimental observation. In order to gain insight into this phase transition continued experimental and theoretical work on the A_d-A_1 transition is needed. The latent heat of the A_d-A_1 transition should vanish at the bicritical point. The present model also indicated the various critical exponents of this transition, which was an open problem.

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